Received: *November 20, 1998*, Accepted: *November 23, 1998*

Rotating black branes wrapped on Einstein spaces

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Abstract: We present new rotating black brane solutions which solve Einstein's equations with cosmological constant Λ in arbitrary dimension d. For negative Λ, the branes naturally appear in AdS supergravity compactifications, and should therefore play some role in the AdS/CFT correspondence. The spacetimes are warped products of a four-dimensional part and an Einstein space of dimension $d-4$, which is not necessarily of constant curvature. As a special subcase, the solutions contain the higher dimensional generalization of the Kerr-AdS metric recently found by Hawking et al.

Keywords: Black Holes, p-branes, Classical Theories of Gravity.

JHEP11(1998)019

Contents

1. Introduction and motivation

The AdS/CFT correspondence [1, 2, 3, 4] relates conformal field theory on a manifold $\mathcal{B}$ to supergravity or string theory on $\mathcal{M} \times \mathcal{K}$, where $\mathcal{K}$ is a compact space, and $\mathcal{M}$ denotes an Einstein manifold with negative cosmological constant, which is required to possess a conformal compactification $\bar{\mathcal{M}}$ with boundary $\partial\bar{\mathcal{M}} = \mathcal{B}$. The conformal field theory partition function, $Z_{CFT}(\mathcal{B})$, is then conjectured to be a sum over all $\mathcal{M}_i$ with $\partial\bar{\mathcal{M}}_i = \mathcal{B}$,

$$Z_{CFT}(\mathcal{B}) = \sum_i Z_S(\mathcal{M}_i)\,, \tag{1.1}$$

where $Z_S(\mathcal{M}_i)$ is the string theory partition function on $\mathcal{M}_i$.

Given this correspondence, it is interesting to look for new bulk supergravity spacetimes representing Einstein spaces with negative cosmological constant, as they furnish important information on the conformal field theory on their boundary by means of (1.1), e. g. information on phase transitions etc.

Of particular interest in this context is the relationship between rotating black objects in the bulk and the conformal field theory on the boundary. This has been examined recently in [5] for Kerr-AdS black holes. (See also [6], where rotating D-branes have been used to study large N QCD). Now already in four dimensions there is a variety of black configurations apart from the Kerr-AdS solution. A class of such objects was presented in [7] as stationary generalizations of so-called topological black holes. In the present paper we shall find higher-dimensional versions of these solutions, generalizing the d-dimensional Kerr-AdS metric recently found in [5] to arbitrary topologies. The found solutions represent, in the broadest sense, rotating black branes which can wrap around Einstein spaces. They are different from the spinning M-branes studied previously in the literature [8, 9]. We expect them to provide an interesting ground to study the AdS/CFT correspondence.

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2. The black brane solution

The d-dimensional Kerr-AdS metric presented in [5], is given by

$$
\begin{aligned}
ds^2 = & -\frac{\Delta_r}{\rho^2}[dt - \frac{a}{\Xi}\sin^2\theta d\phi]^2 + \frac{\rho^2}{\Delta_r}dr^2 + \frac{\rho^2}{\Delta_\theta}d\theta^2 + \\
& + \frac{\Delta_\theta \sin^2\theta}{\rho^2}\left[adt - \frac{(r^2+a^2)}{\Xi}d\phi\right]^2 + r^2\cos^2\theta d\Omega^2_{d-4}\,,
\end{aligned} \tag{2.1}
$$

where $d\Omega^2_{d-4}$ denotes the standard metric on the $(d-4)$-sphere (later we shall see that this is actually too restrictive, because also arbitrary Einstein metrics with positive scalar curvature are allowed for $d\Omega^2_{d-4}$), and

$$
\begin{aligned}
\Delta_r &= (r^2+a^2)(1+r^2l^{-2}) - 2\eta r^{5-d} \\
\Delta_\theta &= 1 - a^2l^{-2}\cos^2\theta \\
\Xi &= 1 - a^2l^{-2} \\
\rho^2 &= r^2 + a^2\cos^2\theta\,,
\end{aligned} \tag{2.2}
$$

where η denotes the mass parameter. In d dimensions the rotation group is $\mathrm{SO}(d-1)$, which has $[(d-1)/2]$ Casimir operators ($[x]$ denotes the integer part of x). Hence one expects that the general spinning black hole solution is characterized by $[(d-1)/2]$ angular momentum invariants [10]. For $d = 5$, a generalization of (2.1) to two rotation parameters was given in [5], but here we shall only consider the case of one angular momentum invariant.

The first thing we note is that we can analytically continue (2.1) in order to obtain hyperbolical rather than spherical solutions. To this aim, we write the metric on the $(d-4)$-sphere as

$$
d\Omega^2_{d-4} = d\chi^2 + \sin^2\chi d\Omega^2_{d-5}\,, \tag{2.3}
$$

and substitute

$$
\begin{aligned}
& t \to it\,, \qquad r \to ir\,, \qquad \theta \to i\theta\,, \qquad \phi \to \phi\,, \\
& a \to ia\,, \qquad \eta \to \eta/i^{5-d} \qquad \chi \to i\chi
\end{aligned} \tag{2.4}
$$

in (2.1), yielding

$$
\begin{aligned}
ds^2 = & -\frac{\Delta_r}{\rho^2}\left[dt + \frac{a}{\Xi}\sinh^2\theta d\phi\right]^2 + \frac{\rho^2}{\Delta_r}dr^2 + \frac{\rho^2}{\Delta_\theta}d\theta^2 + \\
& + \frac{\Delta_\theta \sinh^2\theta}{\rho^2}\left[adt - \frac{(r^2+a^2)}{\Xi}d\phi\right]^2 + r^2\cosh^2\theta d\Sigma^2_{d-4}\,,
\end{aligned} \tag{2.5}
$$

where now

$$
\begin{aligned}
\Delta_r &= (r^2+a^2)(-1+r^2l^{-2}) - 2\eta r^{5-d} \\
\Delta_\theta &= 1 + a^2l^{-2}\cosh^2\theta \\
\Xi &= 1 + a^2l^{-2} \\
\rho^2 &= r^2 + a^2\cosh^2\theta\,,
\end{aligned} \tag{2.6}
$$

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and

$$d\Sigma_{d-4}^2 = d\chi^2 + \sinh^2 \chi d\Omega_{d-5}^2 \tag{2.7}$$

stands for the standard metric on hyperbolic $(d-4)$-space H^{d-4}. One could also take a quotient H^{d-4}/G, where G is a discrete subgroup of the isometry group $\mathrm{SO}(d-4,1)$ of H^{d-4}, acting properly discontinuously. E. g. for $d=6$, we can have a Riemann surface of genus g. Of course, the same is valid also in the spherical case, where also quotients of S^{d-4} are allowed, e. g. Lens spaces or projective spaces.

For $d=4$ the metric (2.5) reduces to the one found in [7] as a stationary generalization of four-dimensional topological black holes. In this case a few comments are in order. First of all, the statement in [7], that the (θ,ϕ) sector can be compactified to obtain Riemann surfaces of genus g by identifying opposite sites of a regular geodesic $4g$-gon centered at the origin $(\theta,\phi)=(0,0)$, is not true. To see this, consider a fundamental region chosen so that the mid-point of one geodesic segment is at $(\theta,\phi)=(\theta_0,0)$. The mid-point of the opposing segment is then at $(\theta,\phi)=(\theta_0,\pi)$. Let us denote these mid-points respectively by p_1 and p_2. Consider the identification map J that takes the first segment to the second segment, and in particular p_1 to p_2. The push-forward of J then takes the set of one-forms $(dt, d\theta, d\phi)$ at p_1 to the set $(dt, -d\theta, -d\phi)$ at p_2. As the coefficient of $dtd\phi$ in the metric is the same at p_1 and p_2, this means that J does not take the metric at p_1 to the metric at p_2, so the metric would be discontinuous. [1] Therefore, as it stands, the metric (2.5) for $d=4$ describes rotating black membranes in AdS space, and only in the static case one can compactify them to obtain topological black holes. For $d>4$, the same remains true for the (θ,ϕ) sector of the spacetime, however now we can compactify the additional space with metric $d\Sigma_{d-4}^2$ as desribed above, without running into difficulties.

Now the metrics (2.1) and (2.5) are not the end of the story. It is known [7] (see also below) that for $d=4$ they emerge as a special case of the most general Petrov-type D metric found by Plebanski and Demianski [11]. This, in turn, contains other black objects as subcases, e. g. a four-dimensional rotating cylindrical AdS black hole [7]. Besides, in view of the fact that the horizon of d-dimensional static topological AdS black holes is not necessarily a space of constant curvature, but can be an arbitrary Einstein space [12], we expect that we can replace the constant curvature metrics $d\Omega_{d-4}^2$ and $d\Sigma_{d-4}^2$ used above by more general Einstein metrics.

The most general known Petrov-type D solution of Einstein's equations in $d=4$ with cosmological constant, found in [11], reads

$$\begin{aligned} ds^2 = \frac{1}{(1-pq)^2} \Bigg\{ & \frac{p^2+q^2}{\mathcal{P}} dp^2 + \frac{\mathcal{P}}{p^2+q^2}(d\tau + q^2 d\sigma)^2 + \\ & + \frac{p^2+q^2}{\mathcal{L}} dq^2 - \frac{\mathcal{L}}{p^2+q^2}(d\tau - p^2 d\sigma)^2 \Bigg\}, \end{aligned} \tag{2.8}$$

[1] D. K. , like the other authors of [7], would like to thank Jorma Louko for pointing out this.

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where the structure functions are given by

$$\mathcal{P} = \left(-\frac{\Lambda}{6}+\gamma\right) + 2np - \epsilon p^2 + 2\eta p^3 + \left(-\frac{\Lambda}{6}-\gamma\right)p^4,$$
$$\mathcal{L} = \left(-\frac{\Lambda}{6}+\gamma\right) - 2\eta q + \epsilon q^2 - 2nq^3 + \left(-\frac{\Lambda}{6}-\gamma\right)q^4. \tag{2.9}$$

Λ is the cosmological constant, η and n are the mass and nut parameters, respectively, and ϵ and γ are further real parameters. (For details cf. [11]). Rescaling the coordinates and the constants according to

$$p \to L^{-1}p\,, \quad q \to L^{-1}q\,, \quad \tau \to L\tau\,, \quad \sigma \to L^3\sigma\,, \quad \Lambda \to \Lambda$$
$$\eta \to L^{-3}\eta\,, \quad \epsilon \to L^{-2}\epsilon\,, \quad n \to L^{-3}n\,, \quad \gamma \to L^{-4}\gamma + \frac{\Lambda}{6}\,, \tag{2.10}$$

and taking the limit as $L \to \infty$, one obtains

$$ds^2 = \frac{p^2+q^2}{\mathcal{P}}dp^2 + \frac{\mathcal{P}}{p^2+q^2}(d\tau + q^2 d\sigma)^2 +$$
$$+\frac{p^2+q^2}{\mathcal{L}}dq^2 - \frac{\mathcal{L}}{p^2+q^2}(d\tau - p^2 d\sigma)^2, \tag{2.11}$$

where now

$$\mathcal{P} = \gamma + 2np - \epsilon p^2 - \frac{\Lambda}{3}p^4,$$
$$\mathcal{L} = \gamma - 2\eta q + \epsilon q^2 - \frac{\Lambda}{3}q^4. \tag{2.12}$$

Setting now

$$q = r\,, \qquad p = a\cosh\theta\,, \qquad \tau = t - \frac{a\phi}{\Xi}\,, \qquad \sigma = -\frac{\phi}{a\Xi}\,,$$
$$\epsilon = -1 - \frac{\Lambda a^2}{3}\,, \qquad \gamma = -a^2, \qquad n = 0\,, \qquad \Lambda = -3l^{-2}, \tag{2.13}$$

one gets our solution (2.5) for $d = 4$, whereas for

$$q = r\,, \qquad p = a\cos\theta\,, \qquad \tau = t - \frac{a\phi}{\Xi}\,, \qquad \sigma = -\frac{\phi}{a\Xi}\,,$$
$$\epsilon = 1 - \frac{\Lambda a^2}{3}\,, \qquad \gamma = a^2, \qquad n = 0\,, \qquad \Lambda = -3l^{-2}, \tag{2.14}$$

one obtains the four-dimensional Kerr-AdS metric. In view of the structure of (2.1) and (2.5) and the results obtained above (like $r^2\cos^2\theta \propto q^2p^2$), we make the following warped product ansatz for the generalized metric in d dimensions

$$ds^2 = \frac{p^2+q^2}{\mathcal{P}}dp^2 + \frac{\mathcal{P}}{p^2+q^2}(d\tau + q^2 d\sigma)^2 +$$
$$+\frac{p^2+q^2}{\mathcal{L}}dq^2 - \frac{\mathcal{L}}{p^2+q^2}(d\tau - p^2 d\sigma)^2 + q^2p^2 d\Omega^2_{d-4}\,, \tag{2.15}$$

where we assume $d\Omega_{d-4}^2$ to be Einstein with scalar curvature $(d-4)\kappa$, with κ a constant. $\mathcal{P}(p)$ and $\mathcal{L}(q)$ are structure functions, which we have to determine. The Ricci tensor of (2.15) can be computed by the following

Proposition [13]*: Let $M = B \times_f F$ be a warped product, with $D = dimF > 1$, X, Y horizontal (i. e. tangent to the basis B) and V, W vertical (i. e. tangent to the fiber F) vector fields. Then*

$$\mathrm{Ric}(X,Y) = {}^B\mathrm{Ric}(X,Y) - (D/f)H^f(X,Y) \tag{2.16}$$
$$\mathrm{Ric}(X,V) = 0 \tag{2.17}$$
$$\mathrm{Ric}(V,W) = {}^F\mathrm{Ric}(X,Y) - g(V,W)f^{\#}\,, \tag{2.18}$$

where $g(.,.)$ denotes the metric on M,

$$f^{\#} = \frac{\Delta f}{f} + (D-1)\frac{g(\mathrm{grad}f, \mathrm{grad}f)}{f^2}\,, \tag{2.19}$$

and Δf and H^f are the Laplacian and Hessian on B, respectively.

For the ansatz (2.15), we choose the following vierbein on the basis B

$$\begin{aligned} e^0 &= \frac{\sqrt{\mathcal{L}}}{\rho}(d\tau - p^2 d\sigma)\,, \qquad e^1 = \frac{\rho}{\sqrt{\mathcal{L}}}dq\,, \\ e^2 &= \frac{\rho}{\sqrt{\mathcal{P}}}dp\,, \qquad e^3 = \frac{\sqrt{\mathcal{P}}}{\rho}(d\tau + q^2 d\sigma)\,, \end{aligned} \tag{2.20}$$

where we introduced

$$\rho^2 = p^2 + q^2. \tag{2.21}$$

The only nonvanishing orthonormal frame components of the Ricci tensor ${}^B\mathrm{Ric}$ of the basis manifold then read

$$\begin{aligned} {}^B\mathrm{Ric}_{00} &= -{}^B\mathrm{Ric}_{11} = \frac{\mathcal{L}''\rho^2 + 2\mathcal{P}'p - 2\mathcal{L}'q + 2\mathcal{L} - 2\mathcal{P}}{2\rho^4} \\ {}^B\mathrm{Ric}_{22} &= {}^B\mathrm{Ric}_{33} = -\frac{\mathcal{P}''\rho^2 - 2\mathcal{P}'p + 2\mathcal{L}'q - 2\mathcal{L} + 2\mathcal{P}}{2\rho^4}\,, \end{aligned} \tag{2.22}$$

where the primes denote derivatives with respect to p or q. For our warping function, $f = pq$, the Hessian is given by

$$\begin{aligned} H^f_{00} &= H^f_{11} = -\frac{(\mathcal{L}'\rho^2 + 2q(\mathcal{P} - \mathcal{L}))p}{2\rho^4} \\ H^f_{22} &= H^f_{33} = \frac{(\mathcal{P}'\rho^2 - 2p(\mathcal{P} - \mathcal{L}))q}{2\rho^4}\,, \end{aligned} \tag{2.23}$$

the off-diagonal components being zero. Let us consider first eq. (2.18). Requiring $\mathrm{Ric}(V,W) = \Lambda g(V,W)$ and ${}^F\mathrm{Ric}(V,W) = \kappa g_F(V,W)$, where $g_F(V,W) = g(V,W)/f^2$

is the fiber metric, and inserting the expressions for the Laplacian and the gradient, one arrives at the equation

$$\Lambda p^2 q^2 \rho^2 = \kappa\rho^2 - qp(\mathcal{P}'q + \mathcal{L}'p) - (D-1)(\mathcal{P}q^2 + \mathcal{L}p^2)\,, \tag{2.24}$$

with $D = d-4$ in our case. Applying the operator $\partial_p^4\partial_q^2$ to (2.24) yields the sixth order differential equation

$$(D+4)\mathcal{P}^{(5)} + p\mathcal{P}^{(6)} = 0 \tag{2.25}$$

for $\mathcal{P}$, which has the solution

$$\mathcal{P} = Cp^{1-D} + \sum_{i=0}^{4} \alpha_i p^i, \tag{2.26}$$

where $C \in \mathbb{R}$ and the α_i are integration constants. Applying $\partial_p^2\partial_q^4$ to (2.24), we get the same differential equation (2.25) for $\mathcal{L}$. This means that we can write $\mathcal{L}$ in the form

$$\mathcal{L} = Bq^{1-D} + \sum_{i=0}^{4} \beta_i q^i, \tag{2.27}$$

B, β_i being integration constants. We now insert the results (2.26) and (2.27) into (2.24) and compare equal powers of $p^i q^j$, which gives for the coefficients of the fourth-order polynomials

$$\begin{gathered}\kappa = (D-1)\alpha_0\,, \qquad \beta_0 = \alpha_0\,, \qquad \alpha_1 = \beta_1 = 0\,, \qquad \alpha_2 = -\beta_2\,, \\ \alpha_3 = \beta_3 = 0\,, \qquad \Lambda = -(3+D)\alpha_4\,, \qquad \beta_4 = \alpha_4\,. \end{gathered} \tag{2.28}$$

One finds that with these coefficients the horizontal equation (2.16) is automatically satisfied, so we have found the final solution

$$\begin{aligned}\mathcal{P} &= Cp^{5-d} + \frac{\kappa}{d-5} + \alpha_2 p^2 - \frac{\Lambda}{d-1}p^4 \\ \mathcal{L} &= Bq^{5-d} + \frac{\kappa}{d-5} - \alpha_2 q^2 - \frac{\Lambda}{d-1}q^4\,. \end{aligned} \tag{2.29}$$

In the special case $d = 5$ (corresponding to fiber dimension $D = 1$), the above proposition is not valid, so one has to carry out a separate calculation, yielding

$$\begin{aligned}\mathcal{P} &= C + \alpha_2 p^2 - \frac{\Lambda}{4}p^4 \\ \mathcal{L} &= B - \alpha_2 q^2 - \frac{\Lambda}{4}q^4\,. \end{aligned} \tag{2.30}$$

Starting from the general metric (2.15) with structure functions given by (2.29), we can recover the d-dimensional Kerr-AdS solution (2.1) by setting

$$\begin{gathered}B = -2\eta\,, \qquad C = 0\,, \qquad q = r\,, \qquad p = a\cos\theta\,, \qquad \alpha_2 = -(1 + a^2 l^{-2})\,, \\ \tau = t - \tfrac{a\phi}{\Xi}\,, \qquad \kappa = a^2(d-5)\,, \qquad \sigma = -\tfrac{\phi}{a\Xi}\,, \qquad \Lambda = -(d-1)l^{-2}\,, \end{gathered} \tag{2.31}$$

with similar formulas for the hyperbolical case (2.5). An interesting feature we notice is that the metric $d\Omega^2_{d-4}$ occuring in (2.1) is not necessarily of constant curvature; it is merely required to be Einstein with positive scalar curvature $(d-5)(d-4)$.

A different solution can be obtained by the choice

$$\begin{array}{lllll} B = -2\eta\,, & C = 0\,, & q = r\,, & p = a\zeta\,, & \alpha_2 = 0\,, \\ \tau = t\,, & \kappa = a^2(d-5)\,, & \sigma = \frac{\phi}{a}\,, & \Lambda = -(d-1)l^{-2}, & \end{array} \tag{2.32}$$

leading to

$$\begin{aligned} ds^2 &= -\frac{\Delta_r}{\rho^2}[dt - a\zeta^2 d\phi]^2 + \frac{\rho^2}{\Delta_r}dr^2 + \frac{\rho^2}{\Delta_\zeta}d\zeta^2 + \\ &\quad + \frac{\Delta_\zeta}{\rho^2}[a dt + r^2 d\phi]^2 + r^2\zeta^2 d\Omega^2_{d-4}\,, \end{aligned} \tag{2.33}$$

where again $d\Omega^2_{d-4}$ denotes the metric of an Einstein space with positive scalar curvature $(d-5)(d-4)$, and

$$\begin{aligned} \Delta_r &= a^2 - 2\eta r^{5-d} + r^4 l^{-2} \\ \Delta_\zeta &= 1 + a^2 l^{-2}\zeta^4 \\ \rho^2 &= r^2 + a^2\zeta^2\,. \end{aligned} \tag{2.34}$$

(2.33) is a d-dimensional generalization of the cylindrical black hole presented in [7] (cylindrical, because the (ζ, ϕ) sector has the topology of a cylinder ($0 \le \phi \le 2\pi$ with endpoints identified, and $\zeta \in \mathbb{R}$)).

We can also recast (2.15) in the canonical form

$$ds^2 = -N^2 d\tau^2 + \frac{\rho^2}{\mathcal{L}}dq^2 + \frac{\rho^2}{\mathcal{P}}dp^2 + \frac{\Sigma^2}{\rho^2}[d\sigma - \omega d\tau]^2 + q^2p^2 d\Omega^2_{d-4}\,, \tag{2.35}$$

where

$$\Sigma^2 = \mathcal{P}q^4 - \mathcal{L}p^4\,, \tag{2.36}$$

and the lapse function N and the angular velocity ω are given by

$$N^2 = \frac{\mathcal{P}\mathcal{L}\rho^2}{\Sigma^2}\,, \qquad \omega = -\frac{\mathcal{P}q^2 + \mathcal{L}p^2}{\Sigma^2}\,. \tag{2.37}$$

Let us now briefly discuss why the found solutions can be regarded in a certain sense as branes. First of all we note that the (θ, ϕ) or (ζ, ϕ) sectors of (2.5) respectively (2.33) are membrane-like,[2] even if the curvature of these membranes is not constant. The $d-4$ additional dimensions form an Einstein space, as we saw above. If this Einstein space is compact, the additional dimensions can wrap around it.

Now, given the new black configurations, one would like to calculate the Euclidean action, as this is needed in the AdS/CFT correspondence according to (1.1) (in the

[2] In the case of (2.33) these membranes are wrapped on a circle parametrized by ϕ.

supergravity approximation). Furthermore, one also needs the Euclidean action, if one wishes to discuss the thermodynamics of the black objects. In this context it would be of interest to examine whether a first law of black hole mechanics is valid. The calculation of the Euclidean action was done in [5] for (2.1), i. e. for the d-dimensional Kerr-AdS black hole. For e. g. (2.5), the thermodynamical discussion is hindered by the fact that the (θ, ϕ) sector of the spacetime is noncompact, and one cannot define quantities like mass or angular momentum *per unit volume* like it is usually done for p-branes with Poincaré invariance on the world volume, as these quantities would still depend on θ. Especially, for $d = 4$ (and this should hold also for arbitrary d), one obtains a mass density proportional to $\sinh\theta$, whereas the angular momentum density goes with $\sinh^3\theta$ [7]. Therefore we do not expect that a first law will be valid for the *densities* of the conserved quantities. If one were able to calculate a finite Euclidean action, these difficulties would have overcome, but to this aim it is not sufficient to subtract a suitably chosen background in order to cancel the divergences in r, because the divergences in θ remain. The calculation of the Euclidean action remains therefore an open issue, which we shall leave for future investigations.

Nevertheless, it is still possible to extract some interesting thermodynamical informations, as we shall see. Let us carry out the analytical continuation $\tau = -i\mathcal{T}$ in (2.35), whereupon we obtain the "quasi-Euclidean" section

$$ds^2 = N^2 d\mathcal{T}^2 + \frac{\rho^2}{\mathcal{L}}dq^2 + \frac{\rho^2}{\mathcal{P}}dp^2 + \frac{\Sigma^2}{\rho^2}[d\sigma + i\omega d\mathcal{T}]^2 + q^2p^2 d\Omega^2_{d-4}\,. \tag{2.38}$$

This metric would be the starting point for a calculation of the Euclidean action [14]. The temperature of the black object can be found by the requirement that no conical singularities appear in (2.38). We assume to have an event horizon on a zero q_H of $\mathcal{L}(q)$, i. e. $\mathcal{L}(q_H) = 0$. Using the expansion

$$\mathcal{L}(q) = \mathcal{L}'(q_H)(q - q_H) \tag{2.39}$$

near $q = q_H$, and defining the new coordinate

$$Q = 2\sqrt{\frac{q - q_H}{|\mathcal{L}'(q_H)|}}\,, \tag{2.40}$$

(2.38) can be written near $q = q_H$

$$ds^2 = \rho_H^2\left(dQ^2 + \frac{\mathcal{L}'(q_H)^2 Q^2}{4q_H^4}d\mathcal{T}^2\right) + \frac{\rho_H^2}{\mathcal{P}}dp^2 + \frac{\Sigma_H^2}{\rho_H^2}[d\sigma + i\omega_H d\mathcal{T}]^2 + q_H^2 p^2 d\Omega^2_{d-4}\,, \tag{2.41}$$

from which we see that the period of $\mathcal{T}$ must be

$$\beta = \frac{4\pi q_H^2}{|\mathcal{L}'(q_H)|} \tag{2.42}$$

in order to avoid conical singularities. (Besides, regularity of the metric also requires to identify $\sigma \sim \sigma + i\beta\omega_H$ [14], where $\omega_H = -1/q_H^2$ is the angular velocity on the horizon).

The Hawking temperature is then

$$T = \beta^{-1} = \frac{|\mathcal{L}'(q_H)|}{4\pi q_H^2} . \tag{2.43}$$

As an example, we calculate (2.43) for the generalized cylindrical black hole spacetime (2.33), obtaining

$$T = \frac{r_H}{\pi l^2} - \frac{\eta(5-d)r_H^{2-d}}{2\pi} , \tag{2.44}$$

where r_H is a zero of Δ_r (2.34), i. e. the location of the event horizon. One observes that for $d > 5$, the temperature (2.44) is always positive; it never becomes zero, as $\eta > 0$ in order to have event horizons. We get a minimal temperature

$$T_{min} = \frac{r_H}{\pi l^2}\frac{d-1}{d-2} \tag{2.45}$$

for

$$r_H = \left(\frac{l^2\eta}{2}(d-5)(d-2)\right)^{\frac{1}{d-1}} . \tag{2.46}$$

This means that the black object contributes to the thermodynamics of the corresponding boundary conformal field theory only if the temperature is high enough, a behaviour similar to the Hawking-Page phase transition [15] from thermal AdS space to the Schwarzschild-AdS black hole, which was studied in the context of the AdS/CFT correspondence by Witten [4]. In the present case, the boundary conformal field theory would live on $T^2 \times \mathbb{R} \times F$, where T^2 is a two-torus, and F an Einstein manifold of dimension $D > 1$ with positive scalar curvature, and due to the minimal temperature necessary for the existence of the black hole in the bulk, we expect a phase transition in this boundary conformal field theory.

The example above indicates that the found solutions may be interesting in the context of the AdS/CFT correspondence; a more precise study of them could reveal further information on conformal field theories.

Acknowledgments

This work has been supported by a research grant within the scope of the *Common Special Academic Program III* of the Federal Republic of Germany and its Federal States, mediated by the DAAD.

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